PERT & CPM

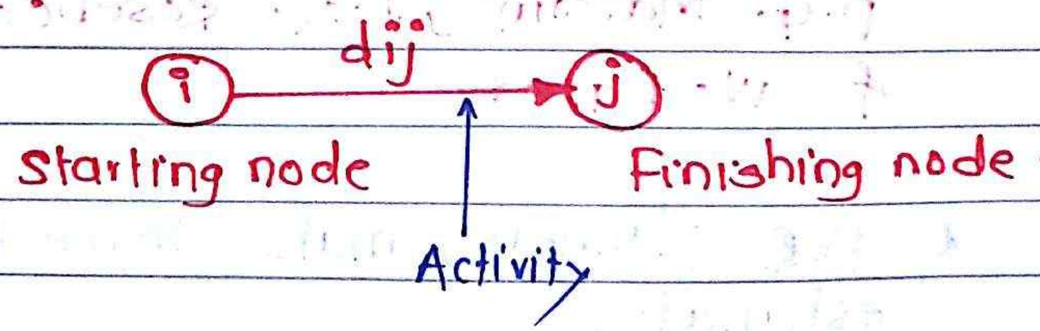
* **Project:-** It is the combination of interelated-ed activities which must be executed in a certain order before the entire task can be completed.

* Project evaluation review technique is used for new projects for which no past data, history is available.

* PERT is event oriented [PES] since slack concept is used

* CPM is activity oriented [CAF] hence float concept is used.

For estimation of time required to complete the project through PERT & CPM we use network diagram which is built by arrow diagram.

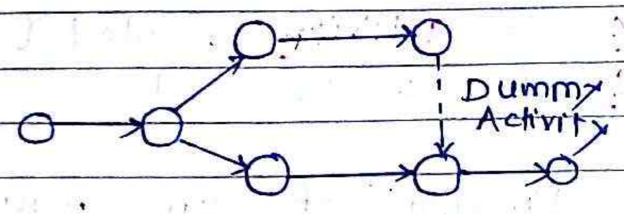


dij ⇒ Duration of activity.

* In network diagram, the nodes should be numbered from left to right by fulcursion rule :

* Dummy Activity :- An activity which does not consume resource or time but it is used to identify the precedence relationship & used only to maintain the logic in network.

i) It is used when two parallel activities would have same tail & head.



* PERT developed by U.S. Navy sponsored research team composed of D.G. Malcom, J.R. Roseboom, C.E. Clerk & W. Fazor.

* PERT planners, make three kinds of time estimates;

1) Optimistic time [t_o] :-

Shortest possible time of an activity when everything is assumed to go well.

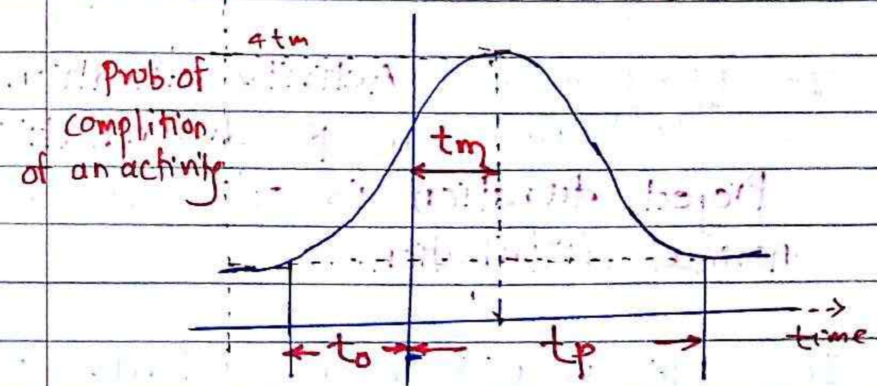
2) Most likely time : [t_m] :-

The time which an activity takes most frequently, if performs no. of times in normal conditions.

3) Pessimistic time :- [t_p] :-

The longest time activity can take when everything goes wrong.

* Combination of these 3 time estimate (t_o, t_m, t_p) represents expected time (t_e), which follows β distribution.



$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

$$S.D = \sigma_5 = \frac{t_p - t_o}{6}$$

$$\text{Variance} = \left(\frac{t_p - t_o}{6}\right)^2$$

* Standard normal variate :- (Z)

$$Z = \frac{t - t_{cp}}{\sigma_{cp}}$$

where t = Due time

t_{cp} = Critical path project duration.

σ_{cp} = S.D. of critical path

$= \sqrt{\Sigma \text{ Variance of cp}}$

for respective z calculate $P(Z)$ from table.

Imp

* In PERT method Activity durations is calculated from β distribution & Project duration is calculated from normal distribution.

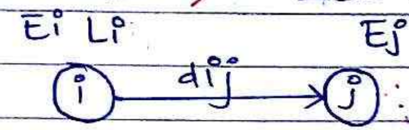
* In CPM method both activity & project durations calculated from normal distribution.

* Critical Path Method :- [CPM] *

* It is developed by M.R. Walker.
* This method is used if project is not new but it has past history available. In this case, only one time estimate is given.
* It is used to predict total project duration.
* Critical path is the longest path in the network from starting event & defines the min. time required to complete the project duration.

* It is activity oriented.
* The duration of time for the activities is assumed as deterministic.

⇒ Critical Activity ⇒ Zero float
⇒ Critical event ⇒ Zero slack.



where E_i ⇒ Earliest start time
L_i ⇒ Latest start time
E_j ⇒ Earliest finish time
L_j ⇒ Latest finish time

* Slack of head ⇒ $L_j - E_j$
* Slack of tail ⇒ $L_i - E_i$

* **Total Float :-**

This represents extra time available for the activity (i-j) without affecting the completion of the project.

* **Free float :-**

It indicates the extra time available for the activity without affecting any other successive activity being started.

* **Independent float :-**

This refers to the extra time available for the activity start to be delayed, independent of without detriment any scheduling decision made else where in the network. This is the time available without affecting values of E & L

Formula :-

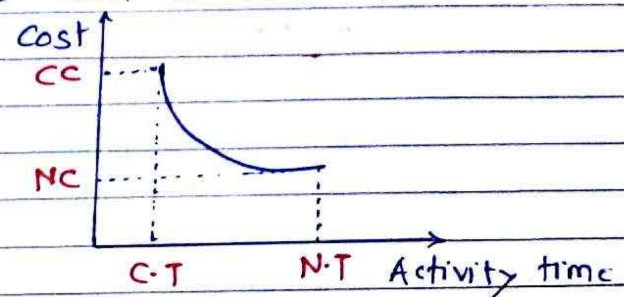
* Total float :- $L_j - E_i - d_{ij}$ [Ext. Values]
* Free float :- $E_j - E_i - d_{ij}$ [E.E]
* Independent float :- $E_j - L_i - d_{ij}$ [Nearest Values]

Imp

* For critical path values of float & slack are zero (0) :

* **Project Crashing / compression :-**

* It is used to reduce project duration.
* Also used to reduce total cost of project



where,

C_c → Crash cost
C_T → Crash time
N_c → Normal cost
N_T → Normal time

$$\text{Cost slope} = \frac{C_c - N_c}{N_T - C_T}$$

Procedure to solve problem :-

1) Find normal critical path.
2) Find cost slope for different activities.
3) Crash the activity which has min. cost slope first to the max. extent possible.

4) Crashing should be done parallely such that other path should not made critical

5) After crashing in max. possible extent find total cost of project

Total cost of project	= N.C. of project + Overhead cost × No. of crashed days + Crashed activities cost (cost slope × days).

Only two variables & 3 constraints possible. But 3 variables & 2 constraints not possibles.

Linear Programming

* Developed by George B. Dantzig.
* LPP deals with the optimization (max. or min.) of a function of variables know as objective function subject to set of linear equations known as constaints. The objective function may be profit, cost, or production capacity.

Requirement of LPP :-

1) Decision variables
2) Well defined objective function.
3) Presence of constraint & in the form of eqn.
4) Non-negativity restriction. → RHS of constraint is non-negative.
   ↳ Decision variables are non-negative.
5) Linearity.

Applications of LPP :-

1) Product Allocation
2) Production planning
3) Inspection
4) Blending
5) Production scheduling
6) Product mix.
7) Transportation.

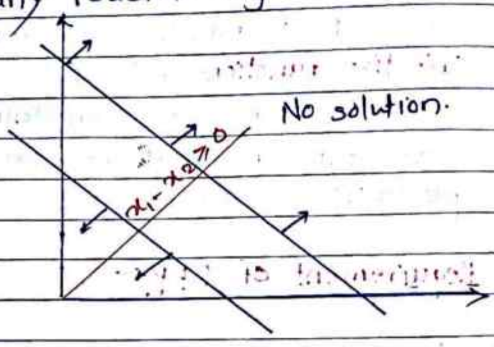
$x_1 - x_2 \leq -1$

change this constraint by multiplying -1

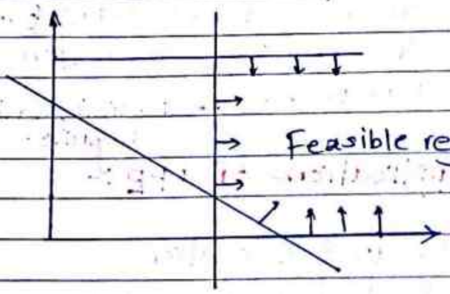
$-x_1 + x_2 \geq 1$

* Special cases :-

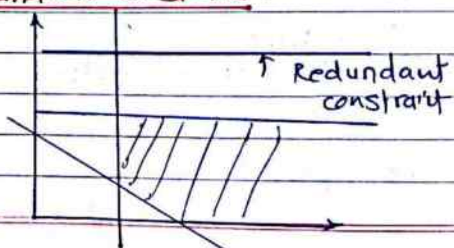
1) Infeasible / No solution :- There is no any feasible region.



2) No limitation of boundary :-



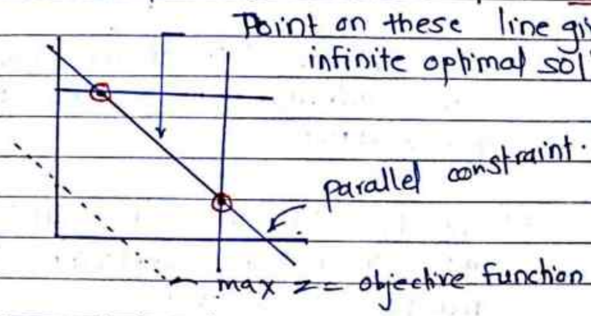
3) Redundant constraint :-



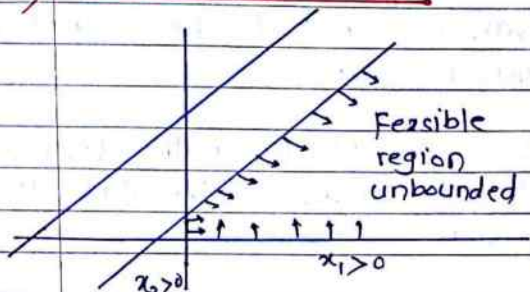
: A constraint which is not a part of feasible region.

4) Multiple optimal / Inifinite optimal sol^n :-

* Objective function should be parallel to atleast one of the constraint & that constraint should be a part of feasible region in form of line; then we said the solution is optimal or infinite optimal sol^n



5) Unbounded solution :-



* Conversion of primal into Dual :-

Note The value of objective function is same for the primal & the dual at the optimum solution. (ie. $z = z'$)

1) If the primal contains n decision variables & 'm' constraints, its dual would be contain 'm' decision variables & n' constraints

2) The maximization of the primal would become minimization problem in the dual & vice versa.

3) The RHS constants of primal would become contribution coefficient in objective function z of dual.

4) The variables in both the problems are non-negative

5) Primal constraints are in the form $\leq$ then the constraints in the dual are $\geq$ & vice versa.

Imp: Dual of a dual is primal

Eg. Primal ⇒

Max $Z = 2x_1 - 3x_2 + x_3$ (→ Min.)

Subject to,

$3x_1 + x_2 + x_3 \leq 5$

$5x_1 + 2x_2 + 2x_3 \leq 10$

$x_1, x_2 \geq 0$ unrestricted

use (=)

Dual ⇒

ie. Min $z = 5u + 10v$

Subject to, $3u + 5v = 2$

$u + v = -3$

$u + 2v = 1$

$u, v \geq 0$

* Degenerate Solution in LPP :-

For LPP problem, corner points of feasible region are as follows, $O(0,0)$, $A(\frac{8}{3}, 0)$, $B(2,2)$, $C(0,6)$ point

If solution is optimal at C. Then we called as degerate solution as, one of the basic variable (x_1, x_2) has zero v

ie. In above problem $C(0,6)$ $x_1 \rightarrow 0$

Queing Theory

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* Analysed by A. K. Earlang
* Study of waiting lines is called queing theory.

* **Arrival pattern** ⇒ Poissons distribution

$$P_n(t) = \frac{e^{-\lambda t} \cdot (\lambda t)^n}{n!}$$

λ = Mean arrival rate
t = Inter arrival rate
n = no. of persons arrived in a time period.
P_n = Prob. that 'n' no. of person's arrived in a time period.

* Denoted by 'M'

* **Service pattern** :- Follows, Exponetial distribution

$$P_n(t) = \mu \cdot e^{-\mu n}$$

mean $= \frac{1}{\mu}$
Variance $= \frac{1}{\mu^2}$
S.D $= \frac{1}{\mu}$

μ = Mean service rate

* Denoted by 'M'

* **Kendall's notation** :- FCFS, LCFS, SIRO, GD → General service discipline [Service in random order]

a / b / c : d / e / f

- a → No. of channels
- d → Queue discipline
- f → Capacity of customers / Population
- a → Arrival rate distribution
- b → Service rate distribution
- e → Capacity of system

*The cost of providing service in a queing system increases with decrease mean time in the queue.

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* **Inter arrival time rate** :- It follows exponential distribution.

$$f(t) = \lambda e^{-\lambda t} \qquad t > 0$$

mean $= \frac{1}{\lambda}$ — Prob. that it will take serviced more than a time $t = e^{-\mu t}$.

Variance $= \frac{1}{\lambda^2}$ — Prob. that customer free within minutes $= 1 - e^{-\mu t}$

S.D $= \frac{1}{\lambda}$

Prob. density function of arrival pattern implies the second one & vice versa.

(M/M/1) : (FCFS/∞/∞)

* **Notations** :-

1) n ⇒ No. of customers in system
2) λ ⇒ Mean arrival rate
3) μ ⇒ Mean service rate
4) L_s ⇒ Length of system
5) L_q ⇒ Length of queue.
6) L_n ⇒ Length of nonempty queue.
7) W_s ⇒ Avg. waiting time in system
8) W_q ⇒ Avg. waiting time in queue.
9) ρ ⇒ Utilisation factor or traffic intensity
10) P_n ⇒ prob. that 'n' customers in system
11) P_0 ⇒ Prob that (zero) customers in system or idle system.
12) W_n ⇒ Average waiting time of an arrival who waits. / Expected waiting time per custo in the system

Avg. length of nonempty queue $L_n = \frac{\mu}{\mu - \lambda}$

Formulae :-

1) $\rho = \frac{\lambda}{\mu}$ It is the proportion of time a server actually spends with customers. % time the typewriter (person) remains in busy.

Prob. that customer has to wait for service is ρ.

2) $P_n = \rho^n P_0$

3) $P_0 = 1 - \rho$ ... Prob. of zero cust. or idle system. Prob. that customer doesn't have to wait for service.

4) $L_s = \frac{\rho}{1-\rho}$ [Avg. no. of customers in the system including those waiting as well as those being served]

5) $L_q = \frac{\rho^2}{1-\rho}$ [Avg. no. of customers in the queue waiting to get service]

6) $W_s = \frac{L_s}{\lambda}$ ... Little's law. [time spent from he arrives & till he leaves]

7) $W_q = \frac{L_q}{\lambda}$ ... Little's law [Avg. time for which a custo. has to wait in the queue to get service]

8) $W_n = \frac{L_n}{\lambda} = \frac{1}{\mu - \lambda}$ Little's law.

9) Prob. that there are atleast K persons in the system $P(n \geq K) = \rho^K$

10) Prob. that there are person's more than K $= \rho^{K+1}$

* For model $(M/M/1) = (FCFS/N/\infty)$
i.e. system capacity is restricted to N

$$\sum_{n=0}^{N} P_n = 1$$

$$P_0 + P_1 + P_2 + P_3 + \dots + P_n = 1$$

$$P_0 + \rho P_0 + \rho^2 P_0 + \dots + \rho^n P_0 = 1$$

$$P_0 [1 + \rho + \rho^2 + \rho^3 + \dots + \rho^n] = 1$$

* For steady state finite length of queue
Service rate > Arrival rate
$\mu > \lambda$

* Balked → If customer decides not to enter the queue since it is too long.

* Reneged → If customer enters a queue but after some time looses patience & leaves it

* Jockeying → When there are two or more parallel queues & the customers move from one queue to the other, they are said to be jockeying.

Transportation Model

* It is special case of LPP.
* For feasible solution Supply = Demand
  i.e. Balanced problem.
* It has $m \times n$ decision variables
* It has $m+n$ constraints.
* It has $(m+n-1)$ basic variables.
  where m = no. of rows
  n = no. of column's

constraints — destinations

Factory	d_1	d_2	d_3	d_4	Supply
F_1	·	·	·	·	
F_2	·	decision variable		·	
F_3	·	·	·	·	
Demand					

Allocations = Basic variables.

* **Degeneracy** :- For basic feasible solution the no. of allocation should be $m+n-1$. If no. of allocations less than $(m+n-1)$ it becomes case of degeneracy.

* **Special case** :- When supply exceeds demand or demand exceeds supply use dummy column or dummy row with zero (0) transportation cost to make the problem balanced.

* **Methods to find initial basic feasible solution :-**

1) North-West corner method
2) Least cost method
3) Row minima method
4) Column minima method
5) Vogels approx. method (VAM)

After finding initial basic feasible solution solve by below two methods,

1) Modified distribution method (MODI)
2) Stepping stone method.

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* Assignment model

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1) It is special case of LPP & transportation problem.
2) Hungerian method is used to solve the problem.
3) For feasible solution no. of rows = no. of column's
4) It has m^2 decision variables.
5) It has 2n constraints.
6) N basic variables.

* Procedure to solve by Hungerian method :-

1) Row reduction ⇒ Substract min. value from all the values of resp. row.
2) Column reduction ⇒ Column sub reduction such that where zero is not present by substracting min. value from other elements.
3) Draw lines horizontal & vertical covering all zero's
4) from uncovered elements select min. value & add it at intersection points & substract from uncover elements.
5) Allocation first done by row wise if single zero then only allocate, else go to the next row. Allocation in column is also similar like rowwise.

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6) Find required cost or solution by adding the allocated position values in given problem.

Sequencing

Sequencing :-

An order or sequence of no. of jobs which require some services.

Assumptions :-

1) All the jobs are ready for processing.
2) Processing time is fixed and deterministic.
3) Set up time is included in processing time.
4) Job travelling time between machines is neglected.

Case I N jobs 1 machine :-

1) FCFS method :- Sequence on First come first serve basis.

2) Shortest processing time (SPT) :- Sequence will be in the increasing order of processing time [min. P.T. first]

3) Earliest Due Date :- [EDD] Sequence will be based on earliest due date i.e. due time.

Jobs	P.T.	Due time	Jobdone	Latness. (-ve) ⇒ 0

(−) : Jobdone − Due time = Latness

Σ time for Jobdone (flow time) Σ Total lateness

1) Total time to complete job is flow time.
2) Avg. flow time = $\frac{\text{Flow time}}{\text{No. of jobs}}$
3) Total lateness (Total tardiness)
4) Avg. lateness = $\frac{\text{Total lateness}}{\text{No. of jobs}}$
5) No. of jobs delayed
6) Max. lateness.

Note SPT minimises flow time & EDD minimise mean lateness.

* If there is tie between two jobs on due date then select the job first which has min. P.T. time.

* If there is tie between two jobs on processing time select the job first which has min. due date

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Case II N jobs 2 machines

Make the optimal sequence on the basis of shortest processing time.

Johnson's rule of optimum. → m/c A | | | | | | m/c B

Jobs (order from optimal sequence)	M/C A In	M/C A Out	M/C B In	M/C B Out	Idle

Σ total m/c A — Σ total m/c B → Make span — Σ Idle time.

* Idle time of m/c A ⇒ Make span − Σ total of m/c A

* Idle time of m/c B ⇒ Σ Idle time

* % utilisation of m/c A = $\frac{\Sigma \text{ total m/c A}}{\text{make span}} \times 100$

* % Utilisation of m/c A = $\frac{\Sigma \text{ total of m/c B} - \text{idle time}}{\text{make span}}$

Shortcut method: If total processing time on m/c A > Total processing time on m/c B then,

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make span = Total P.T on m/c A + Proces. T of last job on m/c B from optimal sequence.

Case III N jobs 3 machines :-

Jobs	m/c A In	m/c A Out	m/c B In	m/c B Out	m/c C In	m/c C Out

$G_1 = A + B$ $G_2 = B + C$

optimal sequence.

G_1 | | | | | | G_2

But it should satisfy following condn.

1) Smallest p. time of m/c A ≥ max. P.T of m/c B

2) Max. P.T on m/c B ≥ min. P.T on m/c C

Shortcut method ⇒ If total P.T on m/c A > Total P.T on m/c C

The make span = Total P.T of m/c A + P.T of last job on m/c B & m/c C from optimal sequence.

Simplex Method :-

Max. $4x + 6y$

Subject to,

$3x + 2y \le 6$

$2x + 3y \le 6$

$x, y \ge 0$

Max. $Z = 4x + 6y + 0S_1 + 0S_2$ ∴ obj. function

to $3x + 2y + S_1 = 6$

$2x + 3y + S_2 = 6$

x_1, x_2, S_1 & $S_2 \ge 0$

C_B: coeff. of S_1 & S_2	C_j / Basic variables	4 / x	6 / y (Enter)	0 / S_1	0 / S_2 ← coeff. of obj. function	Solution (RHS)	Ratio RHS / column element
0	S_1	3	2	0	0	6	$6/2 = 3$
0	leaving S_2	2	3 (key element)	0	1	6 (key row)	$6/3 = 2$ ← least +ve value
	Z_j	0	0	0	0	0	
	$C_j - Z_j$	4	6	0	0		

key column: y; key element: 3

In prob → $z_j - c_j$: -4, -6 (max. value), 0, 0

1) For max. :-

All $c_j - z_j \le 0$ (0 or negative)

2) For min.

All $c_j - z_j \ge 0$ (0 or +ve value)

For second iteration, y → entering variable

S_2 → leaving variable

C_B:	C_j / B.V. (Basic variables)	4 / x	6 / y	0 / S_1	0 / S_2	Solution
0	S_1	$5/3$	0	1	$-2/3$	2
6	y	$2/3$	1	0	$1/3$	$6/3 = 2$
	Z_j	4	6	0	2	
	$C_j - Z_j$	0	0	0	-2	

For S_1 new value

$$= \text{old value} - \frac{\text{Corr. key column value} \times \text{Corr. key row value}}{\text{key element}}$$

$1 = 3 - \frac{2 \times 2}{3} = 3 - \frac{4}{3} = \frac{5}{3}$

$2 = 2 - \frac{3 \times 2}{3} = 0$

$3 = 1 - \frac{2 \times 0}{3} = 1$

$4 = 0 - \frac{2 \times 1}{3} = -\frac{2}{3}$

Solution value S_1: $6 - \frac{2 \times 6}{3} = 6 - \frac{12}{3} = 2$

Case I: On the values of $C_j - Z_j$ & Basic variables we have 3 cases; given on the next page. For above problem

No. of zeros in $c_j - z_j$ row = 3

No. of basic variables $(S_1, y) = 2$

$3 > 2$ ∴ optimal solution has multiple solution ie not uniq

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* **Unique optimal solution :-**
No. of zeros in $c_j - z_j$ row = No. of basic variable (X, Y)

* **Multiple optimal solution :-**
No. of zeros > No. of basic variables.

* **Unbounded solution :-** (-ve ratios)
If there is no least positive ratio i.e. no leaving variable then the problem has an unbounded solution.

key column

		Solution RHS	Ratio
-3		5	-5/3
-2		3	-3/2
0	key row	-6	0/6

There is no least +ve ratio hence Soln Unbounded.

* **Degenerate solution :-**
If there is tie between least +ve ratio i.e. then solution is degenerate for leaving variable.

	RHS	Ratio
4	8	2
2	4	2
1	2	2

Least +ve ratios are equal.

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Critical Ratio Scheduling :- [C.R]

$$CR = \frac{\text{Required date} - \text{Today's date}}{\text{Days needed to complete the job}}$$

or $$CR = \frac{\text{Time remaining}}{\text{Work remaining}}$$

CR < 1 → The job is behind schedule
CR = 1 → The job is on schedule
CR > 1 → The job is ahead of schedule.

* As per this rule least critical ratio are loaded first. (increasing order of CR)

Eg. Two jobs,

	Promised due date	Today's date	work remai days
1	72	60	2
2	65	60	5

$(CR)_1 = \frac{72-60}{2} = 6$ (~~behind~~ ahead of schedule)

$(CR)_2 = \frac{65-60}{5} = 1$ (on schedule)

Job sequence → 2, 1

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* Least slack scheduling :-

Slack = Due date - Processing time

Eg.

	P.T.	D.D.	L.S
A	10	12	2
B	7	13	6
C	2	7	5
D	6	7	1

Sequence → D, A, C, B

Imp * Important Statements about simplex method →

1) A variable which has no physical meaning, but is used to obtain an initial basic feasible solution to the LPP is referred as artificial variable.

2) In simplex method, if during an iteration, all ratios of right hand side of bi to the coefficient of entering variable are found to be negative ie. -ve ratios, it implies that the problem has unbound solution.

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3) A tie for leaving variable in simplex procedure implies Degeneracy. ie. when two min. +ve ratios of RHS to the coefficient in the key column are equal. Equal or tie between ratios → Degeneracy.

4)* In simplex method the min. positive ratio of RHS to the coefficient in the key column decides the leaving variable

BREAK EVEN ANALYSIS AND PRODUCTION COST CONCEPTS

Cost →: Cost is the amount of resources sacrificed or given upto achieve a specific objective which may be the acquisition of goods or services. Cost are always expressed in money terms.

Classification of costs →

1) Direct Material → cost of material which become a major part of the finished product. They are the raw matt. that become integral part of the finished product. eg. Raw cotton in textiles, Crude oil to make diesel.

2) Direct labour → labour associated with workers who are engaged in the production process.
eg. labour of machine operators, assembly operators.

3) Direct expenses → expenditure other than direct matt & direct labour are called direct expenses or chargeable expenses.
eg. cost of special layout, design or drawing, hiring special m/c for specific product.

4) Factory Overheads → / Manufacturing cost: includes cost of indirect material / indirect labour & indirect expenses.

i) Indirect labour → which will not affect the composition or construction of product. eg. foreman, shop clerks, maintenance employees.

ii) Indirect matt. → It is needed but it is not possible to trace or identify with end product eg. Cutting oil, lubricants.

iii) Indirect expenses are the expenditure incurred by the manuf. company from the beginning of production to its completion & transfer to finished goods store.

5) Distribution & administrative overheads →

i) Distribution → Marketing & selling Overheads like advertising, salesman salaries & commission packaging, storage transportation.

ii) Administrative → includes cost of planning & controlling of general business operations. eg. chairman's salary, fee of board directors, rent of administrative office.

All costs which are not charged to production and sales are included in administrative overheads.

1) Direct Material + Direct labour + Direct expenses = Prime Cost.

2) Indirect material + Indirect labour + Indirect expenses = Factory overheads.

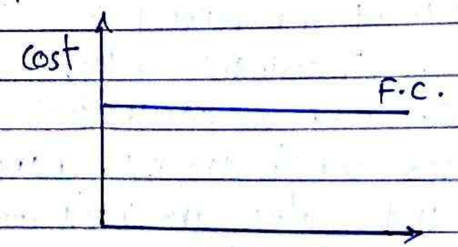
3) Prime cost + Factory overheads = Factory cost / production cost.

4) Factory cost + Distr. & administrative overhead = Total cost.

5) Selling cost = Total cost + Profit

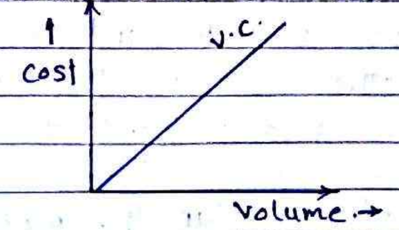
2) Classification based on Activity or volume →

A) Fixed cost → The cost which do not change for a given period in spite of change in volume of production. Independent of volume of production.
eg. Rent, taxes, salaries of supervisors, depreciation, insurance etc.

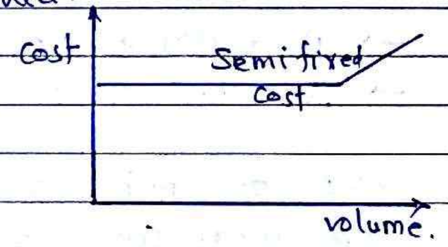


F.C. are constant upto specific volume or range of volume.

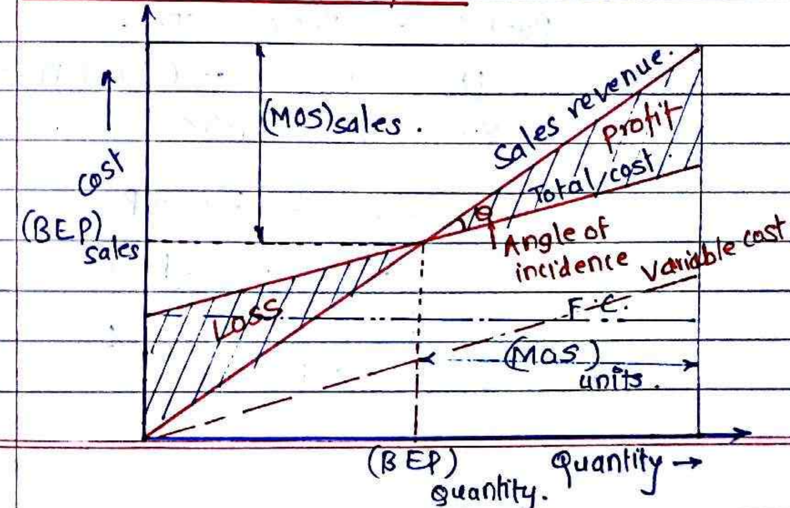
B] Variable cost → These vary directly & proportionately with volume of production.
eg. Direct matt & Direct labour costs.



C] Mixed costs → Made up of fixed & variable costs. Because of v.c they fluctuate with volume. It is basically variable but whose slope may change abruptly when a certain level is reached.



* Break Even Analysis [BEP]



→ Establishes relationship among the factors affecting profit.
→ Effect of changes in volume on profit.
→ Helps to understand the effect of alternative decisions that convert costs from variable to fixed the costs which increase sales volume & revenue.
→ It helps to plan the profit.

Assumptions →

→ Selling price constant [No. discount]
→ Linear relation between sales & costs.
→ F.C do not vary with quantity only V.C will vary.
→ Production & sales are equal [No inventory]
→ No other factors affect except quantity.

BEP at which sales volume revenue equal to the total cost. i.e. profit is zero.

At BEP Profit = 0

$$\text{Profit} = T.SC - T.C.$$

$$P = n.S.C - (Fc + n\,V.C)$$

$$n\,S.C = Fc + n.Vc$$

$$n(SC - Vc) = Fc + P$$

$$\boxed{n = \frac{F.C + P}{SC - V.C.}}$$

At BEP → P = 0

$$n = \frac{F.C. + P}{\text{Contribution Margin}} \qquad n.(CM) = F.C + P.$$

* **Contribution Margin →** Diff. between total S.C & total variable cost. It is also known as marginal profit or Gross margin.

* **Profit / volume [P/V] ratio →** It measures profitability in terms of sales.

$$\frac{P}{V} = \frac{S.C - V.C}{S.C} = \frac{CM}{S.C} = \frac{F.C + P}{S.C.}$$

* **Margin of safety →**

i) sales wise → It is difference of o/p at any point compare to o/p at BEP

$$(MOS)\ \text{sales} = (\text{Sale})_x - (BEP)\ \text{sales}.$$

$$= Sx - F.C/S$$

$$= S.x - \frac{F \times S}{S - V.C}$$

$$= S\left[\frac{x(S - V.C) - F}{S - V.C}\right]$$

$$(MOS)\ \text{sales} = \frac{P}{\frac{S - V.C}{S}} = \frac{P}{(P/V)\ \text{ratio}}$$

ii) (MOS) percentagewise → S

$$(MOS)\ \% = \frac{(\text{Sales})_x - (BEP)\ \text{sales}}{(\text{sales})_x} \times 100$$

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Formulae →

1) Contribution = sales × $\frac{P}{V}$ ratio

2) $BEP = \frac{F.C}{P/V\ ratio}$

3) $(Sales)\ units = \frac{F.C + Profit}{Contribution}$

* **Angle of incidence →** This is angle at which the sales line cuts the total cost line. The management aims at larger angle of incidence indicates high profit rate.

Methods of lowering BEP → Every org. aims at lower BEP so that their fixed costs are recovered & soon the profit begins after the sales starts exceeding BEP.

1) Reduce Fixed cost
2) Reduce the variable cost.
3) Increase the slope of the income line.

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Inventory Control →

Inventory → Materials in stock / idle resource of an enterprise.

Types →

1) Raw material
2) Bought out parts.
3) Work in process inventory.
4) Finished goods inventory.
5) Maintenance, repairs and operating stores.
6) Tools inventory.
7) Anticipation inventory → Change in demand drastically and in case of shut down period for future need.
8) Transportation inventory.

Reasons to keep inventory →

1) To stabilise production
2) To take advantage of price discounts.
3) To meet demand during procurement period.
4) To prevent loss of orders.
5) To keep pace with changing market conditions.

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Inventory control → planned approach of determining what to order, when to order & how much to order and how much to stock so that costs associated with buying and storing are optimal without interrupting production & sales.

Inventory Costs →

1) Purchase cost / Nominal cost → The value of an item is its unit purchasing cost.

2) Capital cost → The amount invested in an item, is an amount of capital not available for other purchases.

3) Ordering cost → / setup cost → / Procurement OR replenishment cost → Acquisition cost → cost of ordering is the amount of money expended to get an item into inventory.

4) Inventory carrying or holding cost → These are the costs associated with holding a given level of inventory on hand & this cost vary in direct proportion to the amount holding & period for holding the stock.

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5) Shortage Cost → When there is a demand for the product and the item needed is not in stock, then we incur a shortage cost or cost associated with stock out. Also known as backorder cost.

Terminology →

1) Demand → It is the no. of items (product required per unit time. It may be deterministic or probabilistic.

2) Order cycle → The time period between two successive orders.

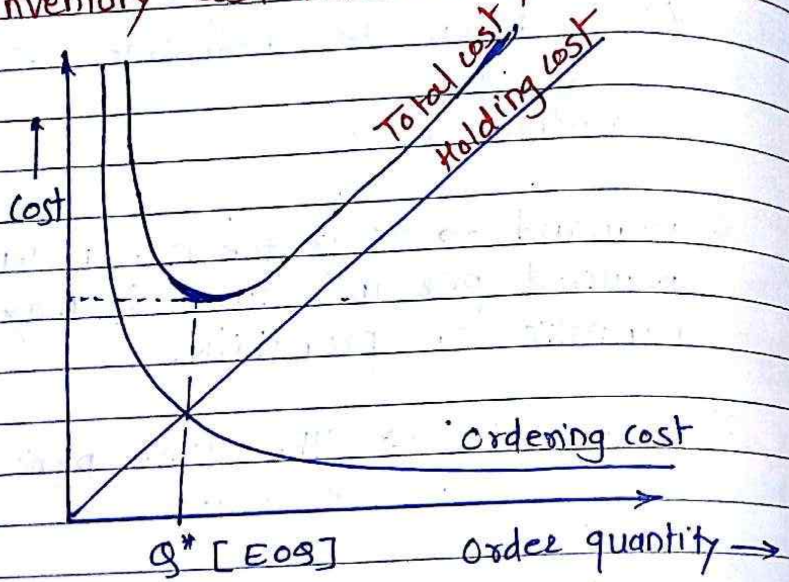
3) Lead time → The length of time between placing an order & receipt of items is called lead time.

4) Safety stock → buffer or min. stock to account for delays in matl. supply & account for sudden increase in demand due to rush orders.

5) Reorder level [ROL] → It is the point at which procurement action is initiated i.e. order placing.

6) Re-order quantity → quantity ordered at reorder point normally this is EOQ.

Inventory cost relationship →



Holding / Annual inv. carrying cost is directly proportional to quantity. Holding cost decreases if the quantity ordered per order is small. The right quantity to be ordered is one that strikes a balance between the two opposing costs. This quantity is referred to as economic order quantity.

Inventory models →

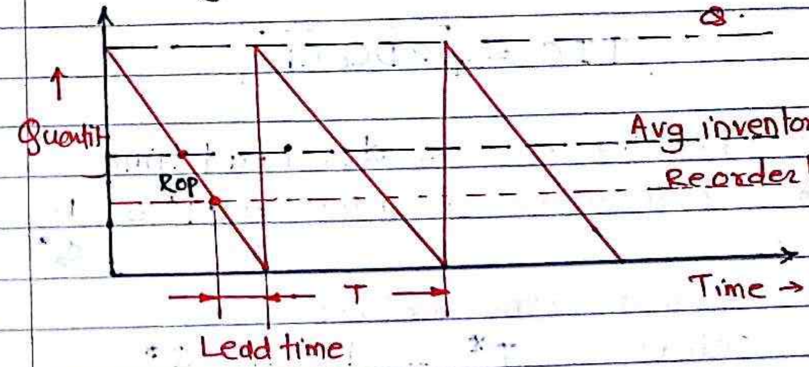
1) Deterministic models → In these model demand & lead time are constant & we need not to carry safety or buffer stock.

2) Probabilistic models → In these models dem. and & lead time are variable so we need to carry safety or buffer stock to prevent stockout.

1) Harris / Wilson model

Model 1 [Basic inventory model] with instantaneous stock replenishment →

1) Demand constant, lead time is zero.
2) quantity discounts are not allowed.
3) Ordering cost doesn't vary with order quan



Q ⇒ Quantity to be ordered at each order point units/order.

C_p ⇒ Unit price of inventory item Rs/

C_o ⇒ Ordering cost/order.

$ch \rightarrow$ Cost of holding inventory item Rs/unit/year.

* Procurement Cost $= D \times C_p$
* Ordering cost $= D/Q \, C_o$
* Holding cost $= Q_{avg} \times ch = \frac{Q}{2} \times ch$
* Total cost $= D \times C_p + \frac{D}{Q} C_o + \frac{Q}{2} C_h$
* $TIC = \frac{D}{Q} C_o + \frac{Q}{2} C_h$

$$\frac{dTIC}{dQ} = 0 = \frac{-DC_o}{Q^2} + \frac{C_h}{2}$$

$$\boxed{Q^* = \sqrt{\frac{2DC_o}{C_h}}}$$

$$\boxed{TIC = \sqrt{2DC_oC_h}}$$

* Reorder level $= d \times$ Lead time
* Optimum no. of orders $= N^* = \frac{D}{Q^*}$
* Optimum time between two successive orders $T^* = \frac{1}{N^*} = \frac{Q^*}{D}$

Note When holding cost is given in terms of holding rate of unit purchase price.
$\therefore$ [I = % per month the $ch = I\% \times 12$ ie. Rs/unit/year]

Robustness or model sensitivity →

Model sensitivity defined as the ratio of total inventory cost at any point to the min. inventory cost corresponding to EOQ.

$$= \frac{TIC(Q)}{TIC(Q^*)}$$

At EOQ ordering cost = Holding cost — ①

$$TIC(Q^*) = \frac{D}{Q} C_o + \frac{Q}{2} ch$$

$$= 2\frac{D}{Q^*} C_o \quad \text{... from eq}^n \text{ ①} \quad — ②$$

$$TIC(Q) = \frac{D}{Q} C_o + \frac{Q}{2} C_h$$

But $Q = KQ^*$

$$= \frac{D}{KQ^*} C_o + \frac{KQ^*}{2} ch$$

$$TIC(Q) = \frac{D}{KQ^*} C_o + K\left[\frac{D}{Q^*} C_o\right]$$

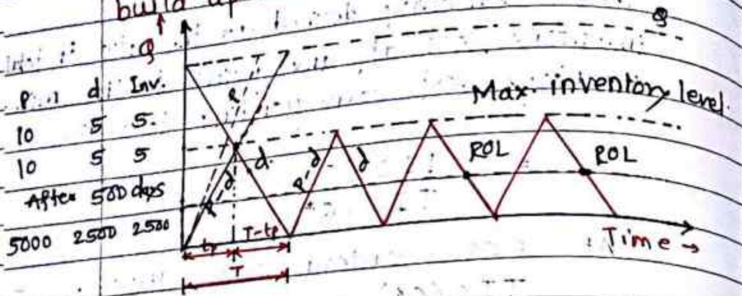
$$TIC(Q) = \frac{D}{Q^*} C_o \left[K + \frac{1}{K}\right] \quad — ③$$

From eq^n ② & ③

$$\frac{TIC(Q)}{TIC(Q^*)} = \frac{\frac{D}{Q^*} C_o \left[K + \frac{1}{K}\right]}{2\frac{D}{Q^*} C_o}$$

$$\boxed{\frac{TIC(Q)}{TIC(Q^*)} = \frac{1}{2}\left[K + \frac{1}{K}\right]}$$

2) Model - 2 [Production model / Continuous build up model]



In this model inventory buildup is gradual rather than instataneous. If P (production rate) d (demand / consumption rate) then total inventory cycle (T) is divided in 2 parts 1) tp (production or manufacturing time) 2) (T−tp) It is the time during which there is no production & only consumption with demand rate d units.

Let Total inventory cost = ordering cost + Holding cost

$$TIC = \frac{D}{Q} C_o + Q_{av} \times C_h$$

$$Q = t_p \times P \qquad t_p = \frac{Q}{P}$$

$$Q_m = (P-d) \times t_p$$

$$Q_m = \left(\frac{P-d}{P}\right) \times Q$$

Q max. Inventory level

$$TIC = \frac{D}{Q} C_o + \frac{Q_m}{2} C_h$$

$$TIC = \frac{D}{Q} C_o + \frac{Q}{2} \left(\frac{P-d}{P}\right) C_h$$

$$\frac{dTIC}{dQ} = 0 = -D C_o \frac{1}{Q^2} + \frac{P-d}{2P} C_h$$

$$\frac{D C_o}{Q^2} = \frac{P-d}{2P} C_h$$

$$Q^* = \sqrt{\frac{2 D C_o}{C_h} \left(\frac{P}{P-d}\right)}$$

$$TIC(Q^*) = \sqrt{2 D C_o C_h \times \left(\frac{P-d}{P}\right)}$$

Model-3 Shortage / Backorder model →

In this model shortages are allowed. Plan shortage or backorder is a situation in which a customer places an order & finds that inventory is out of stock. Shortage results in decrease in net stock in inventory thus reducing cost of inventory. Shortages increases the cycle time & thus lowers the ordering cost. Let S is no. of units backordered, Cb is backorder / shortage cost per unit / year.

Total inventory cost = Ordering Cost + Holding cost + Shortage Cost / stockout cost

$$TIC = \frac{D}{Q} C_o + \frac{(Q-S)^2}{2Q} C_h + \frac{S^2}{2Q} C_b$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{C_h+C_b}{C_b}\right)}$$

$$TIC(Q^*) = \sqrt{2DC_oC_h}\left(\frac{C_b}{C_h+C_b}\right)$$

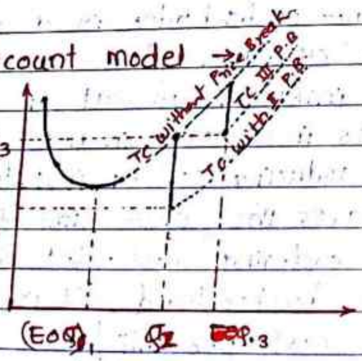
* Optimum no. of units backordered

$$S^* = Q^*\left(\frac{C_h}{C_b+C_h}\right)$$

* Max. inventory level /Safety stock $M^* = Q^* - S^*$

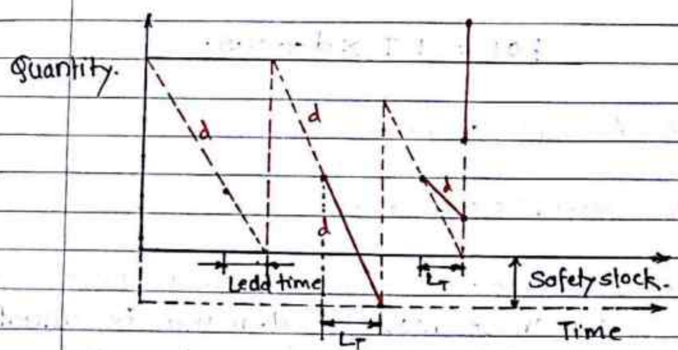
$$= Q^*\left(\frac{C_b}{C_b+C_h}\right)$$

Model-4 Discount model →



In some situations discounts are offered on unit purchase price of inventory and these discounts tell form of P.B discount is always offered on unit purchase price of inventory, so in these model purchasing cost $(D \times C_p)$ also considered along with ordering cost & inv. holding cost in determine economic order size. In these model first of all physible EOQ is computed & then total cost is calculated at physible EOQ is in the given range & total cost also calculated at the next P.B points the order size at which total cost is minimum is the best order size.

2) Probabilistic Models →



Demand & lead time fluctuates they need not to be constant. In these model we carry safety or buffer stock in order to prevent stockout during unfavourable condition.

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Factors encouraging higher safety stock

1) When demand variation is more.
2) Lead time fluctuation is high.
3) When inventory carrying cost is less & is not of much concerned.
4) When stock out caused or loss due to stockout is very high when no. of orders in a year are more & to provide better service level to customer at short notice.

1) ROL = Avg. demand during lead time + Safety stock

$$ROL = LT \times d + S.S.$$

2) Avg. inventory $= \frac{EOQ}{2} + S.S.$

3) Safety stock Cost $= S.S \times Ch$

Model 1 → Static inventory / Demand profit model →

In these models demand is uncertain & decision is based on single order. The model is applicable for perishable items like vegetables, fruit, flowers etc. or for those items which becomes outdated very fast like newspapers. This model is also known as Newspaper boy or Kristmas tree model.

In these model either replacement orders are not possible or becomes very expensive.

Let D → Demand of inventory item.
S → Supply or quantity to be ordered.
c → Profit per unit.
l → Loss per unit.

1) Overdemand / undersupply → $D > S$
Loss $= (D - S) \times$ profit.

2) Oversupply / underdemand → $S > D$
Loss $= (S - D) \times$ loss.

3) Order quantity S

$$P(S-1) < \frac{P}{P+l} < P(S)$$

Where $P(S-1)$ = Cumulative probability of the demand for $(S-1)$ units
$P(S)$ is the cumulative prob. of demand S units.

$$P(S-1) < \frac{Cb}{Cb + Ch} < P(S)$$

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Model-2 Service level OR Safety stock model →

In these model amount of safety stock is kept according to the level of service management aims/wants to achieve.

1) Service level = $\frac{\text{No. of units supplied without delay}}{\text{Total no. of units demanded}}$

2) Vary between 0 – 1 ie. 0 – 100 %.

3) 95 % service level would means that 95 % of customer orders are fulfilled & 5 % of orders rejected due to stockout during lead time. Demand during lead time may be approximated by a normal distribution curve mean of $\bar{x}$ or μ & standard deviation σ. Then the reorder level given by.

$$ROL (x) = \bar{x} + z\sigma$$

z → standard normal variate whose value depends on level of service management wants to achieve.

Z	Service level %
0.84	80 %
1.28	90 %
1.645	95 %
2.33	99 %

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$$(ROL)_x = \bar{x} + z\sigma$$
$$= ADDLT + SS$$

$$\text{Safety stock (S.S)} = z\sigma$$

If demands are $d_1, d_2, d_3, \dots d_n$ then safety stock ⇒

$$S.S. = \text{Max.}[d_1, d_2, d_3 \dots d_n] - \frac{d_1 + d_2 + d_3 + \dots + d_n}{n}$$

If lead time is varying $LT_1, LT_2 \dots LT_n$ then safety stock

$$S.S = \left[\text{Max. of }(LT_1, LT_2 \dots LT_n) - \frac{(LT_1 + LT_2 + \dots LT_n)}{n}\right] \times d$$

For a first half cycle S.D = σ_1, for next half cycle σ_2, then

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$$

1) Avg. inventory = $\frac{EOQ}{2} + z\sigma$

2) Safety Stock Cost = $z\sigma \times C_h$

ABC Analysis → 80-20 law/ Pareto principle.

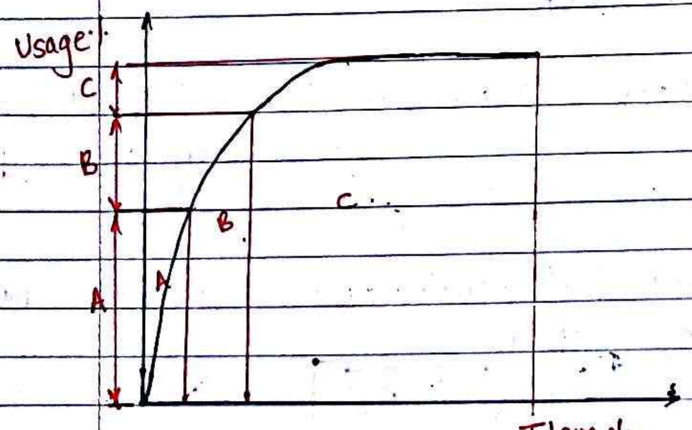
[Always Better Control]

	Usage %	Item %
A	50-60 %	10-20 %
B	30-40 %	30-40 %
C	10-20 %	50-60 %

Product	Unitprice (a)	No. of units to be used (b)	Usage Value (a×b)
1			
2			
n			

$C = \Sigma (a \times b)$

Usage % $= \frac{ab}{C}$



In these system inventory items classified on the basis of usage value of inventory items in ~~inventory~~ monetory terms. Inventories are classified in A, B & C categories for A category items inventory kept is very low. or almost nil & frequent review is done on the other hand for C category items large inventory is kept & it is reviewed after long duration.

2) VED → Vital, Essential, Desirable
It depends on criticality of components.

3) HML Control → High, medium & Low
classified on unit purchase price of inventory item.

4) SDE [Scarce, Difficult, Easily available
classified on easiness of difficulty of availability.

[SOS] Analysis.

5) Seasonal / off seasonal → Based on seasonal variation.

6) FSN Analysis → Fast moving, Slow moving & Non moving.
Requirement Regularly / occasionally / rarely

7) XYZ Analysis → High inventory values → X, Low inventory values → Z

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* Fixed quantity system [Q system]

→ Perpetual inventory system.

→ Order quantity fixed & ordering time varies according to the fluctuation in demand.

→ Reorder normally equals to EOQ.

→ Depending upon the demand, the time interval of ordering varies.

→ Replenishment action is initiated when stock level falls to Reorder level.

→ Safety stock is maintained to account for increase in demand during lead time.

Advantages →

1) Suitable for low unit cost & high volume items.
2) Normally preferred when supplier puts min. quantity restriction.
3) Stock out control will be accurate as the replenishment action is initiated soon after the stock reaches ROL.
4) Appropriate for variety of inventory maintained within the organization.

Limitations →

1) There will be a load on the re-ordering system if many items reach R.O.L. at the same time.
2) The stock levels records and usage rate data are to be maintained.

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* Q - System works like two bin system

1) 1 Bin → Contains the quantity i.e. order quantity = (Q - LC) to satisfy demand between the lead time [Lead time consumption]

2) 2 Bin → Quantity equal to ROL (M+LC) to satisfy demand during replenishment period.

* Periodic Review system / fixed period / P system

→ Fixed ordering interval.

→ Size of the order quantity may vary with changes in demand.

→ Inventory position is verified at pre-fixed interval (weekly / monthly / quarterly)

→ Stock is reviewed at periodic interval & the quantity Q which will be bring the inventory to max. level is ordered.

→ Suitable for high unit cost & less no. items

→ Preferred when supplier delivers at fixed period.

Q = Max. stock − (Stock on hand + stock on order

DEMAND FORECASTING

Before making an investment decision many questions will arise like,

1) what should be the size or amount of capital required.
2) How large should be the size of the workforce?
3) What should be the size of the order & safety stock.
4) what should be the capacity of the plant

The answers for above questions depends upon the forecast for the future level of operations.

Forecasting → Defined by American marketing association, An estimate of sales in physical units (or monetory value) for the specified future period under proposed marketing plan or programme and under the assumed set of economic and other forces outside the organisation for which the forecast is made.

Need for demand forecasting →

1) Majority of activities of industries depend upon future sales.
2) To schedule production activity to ensure optimum utilisation of plants capacity.
3) To prepare material planning to make material available at right quantity and right time.
4) Projected demand for the future assist in decision making w.r.t. investment in plant and machinery, market planning & programme.
5) Provides future trend which is very much essential for product design & development.

Classification

1) Judgemental techniques.
2) Time series methods.
3) Casual methods [Econometric forecasting]

Commonly used techniques in business and industries. Subjective method where in there is a heavy reliance on the past experience of the person & skill.

1) **Opinion Survey Method →** Opinions are collected from the prospective buyers regarding, why they buy a perticular product, what they expect from product.

2) **Executive opinion method →** experts opinion is sought on the future demand for the product. Accuracy depend upon the skill, expertise and experience of the person making the forecast.

3) **Customer and distributor Surveys →** The questionnarie can be given along with the guarantee cards. Estimates of expected sales (distributor surveys can be requested from retail outlets & company's sales force.

4) **Marketing trials →** for new products In this case it is advisable to expose the product to the limited trial. Such a trial is like a controlled experiment in which the market area & the method of presentation

5) **Market Research →** Used for new product as well as existing products. Usually the work is assigned to ext. marketing agencies.

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Purpose of research to gather information regarding the nature of consumption.

6) Delphi technique → A panel of experts are asked sequential questions in which the response to one questionnaire is used to produce next questionnaire. The information available to some experts are made available to the other experts. This technique is an iterative process. It is qualitative Method. Each questionnaire demands a written opinion about sp. subjects & reasons for justifying the opinions from selected technical fields.

Time Series Analysis → The time series analysis consists of determining the trend underlying the demand & extrapolate the future trend.

Trend → (T) → The long period tendency of the data to increase or decrease is referred to as a secular trend. It is long run historical component of the time series which indicates overall growth or decline of the business overtime.

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Cyclic Fluctuations (C) → The magnitud timing & pattern of cyclic fluctuations vary so widely & the awareness of th effect of cyclic variation should influence the forecaster.

Seasonal Variations (S) → Some degree of regularity in a span of one year and are annually repetitive. eg: Diwali festivals, sale of umbrella in mansoon.

Irregular variations (R) → occure without any fixed pattern. These are chance variations that are not explained by trend, cyclic or seasonal variations. There may account for natural calamities lik flood, draught, earthquake etc.

Y [forecasted value] = TCSR

* Moving Average [MA] Forecasting →
Moving avg. is a simple statistical metho to extrapolate and establish trend of past sales. This method uses a past data & calculates a rolling average for a constant period At each period, fresh average is computed at the end of each

period by adding the demand of the most recent period & deleting the data of the old-period since the data in this method changes from period to period, it is called moving average method.

$$MA = \frac{\text{Sum of demands for periods}}{\text{Chosen number of periods}}$$

For the time series values : $D_1, D_2, D_3 \cdots D_n$ the moving avg. for n periods.

$$MA = \frac{D_1 + D_2 + D_3 + \cdots + D_n}{n}$$

* For 3 months MA has a weightage of $1/3^{rd}$, & 5 months MA $1/5^{th}$.

Note The value of n depends upon the speed at which the pattern of demand changes. If the demand pattern is stable a high value of n is selected. If the pattern is not stable a small value of n should be selected.

* Weighted Moving Average →

Sometimes the forecaster wants to use a moving avg. but does not want all the n periods equally weighted as in simple MA method.

In simple MA, equal weightage is given to 1^{st} month, 2^{nd} month & 3^{rd} month in a 3 month moving average. But the org. wants to attach more weightage to the 3^{rd} month & least to the first month.

* Exponential Smoothning Method →

One of disadvantages of the moving avg. forecasting is the laborious operation of maintaining the data for all the prev. years. Expo. Smoothning method requires only the current demand & the forecasted demand.

Demand for the most recent data is given more weightage & the weights assigned to older periods decreases exponentially.

Forecast for the period [Ft] = forecasted demand for the last period + α [Act. demand for the last period − Forecasted demand for the last period]

$$F_t = \alpha D_{t-1} + (1-\alpha) F_{t-1}$$

$$F_t = F_{t-1} + \alpha [D_{t-1} - F_{t-1}]$$

$$F_t = F_{t-1} + \alpha [\text{Error}]$$

where α = Smoothning Constant

$$= \frac{2}{n+1}$$

Note 1) for initial period forecasted value is not given take the demand for the first period = forecasted value for first period

$$F_1 = D_1$$

2) Take the avg. demand values for the forecasted value for the first period & proceed.

$$F_1 = \frac{D_1 + D_2 + D_3 + \cdots + D_n}{n}$$

* **Responsiveness & stability →**

It indicates that forecast have fluctuating or swinging pattern. This are prefered for new product & for that no. of periods are kept low.

$\alpha \rightarrow 0$ to 1

$n \rightarrow 1$ to ∞

Stability → It indicates the forecast pattern have levelled or less fluctuations. It indicates idle situations achieved & used for old existing product & for that no. of periods kept large (∞)

Note If $\alpha = 0$ $n \rightarrow \infty$ in that case

$F_t = F_{t-1}$ ..... Limit of stability.

If $\alpha = 1$ $n \rightarrow 1$ in that case

$F_t = D_{t-1}$ ..... Limit of responsiveness

Note For stable curve the value of α is less.

* **Least Square Method [Regression Analysis] →**

This method is called least square method as the sum of the square of the deviations of the various points from the line of best fit is minimum or least. It gives the eqⁿ of the line for which the sum of the squares of vertical distances between the actual values & the line values are at min.

$$y = a + bx$$

$b \rightarrow$ Slope of the line

$a \rightarrow$ y intercept

Constants a & b are determined by the 2 simultaneous eqn:

$$\Sigma y = Na + b\,\Sigma x$$
$$\Sigma xy = a\,\Sigma x + b\,\Sigma x^2$$
} Normal eqn.

To compute the values of a & b

i) Calculate the deviation (x) for each period & also the sum of deviations.
ii) Find the value of Σx^2
iii) Find the value of Σxy
iv) Calculate the values of a & b
v) Make the sum of deviations $\Sigma x = 0$

$$d = \frac{\Sigma y}{N}$$

$$b = \frac{\Sigma xy}{\Sigma x^2}$$

Note 1) If the time series consists of odd number of years to make $\Sigma x = 0$, the middle value of the time series is taken as the origin.

2) If the time series consists of even no. of years the midway period between two middle periods is taken as origin to make $\Sigma x = 0$

Forecast Error → It is essential to have a good measure of effectiveness of forecasting methods.

Forecast Error → Difference between forecasted demand and the actual demand.

* MAD → (Mean absolute deviation)
It measures avg. forecast error without deviation direction.

$$MAD = \frac{\text{Sum of absolute value of forecast error for all periods.}}{\text{No. of periods.}}$$

$$= \sum_{j=1}^{n} \frac{\text{Forecasted demand} - \text{Actual demand}}{n}$$

$n \Rightarrow$ No. of periods.

MAD → Measured without considering sign. Thus it express the magnitude but not the direction of error.

* BIAS → Measures the forecast error with regard to direction and shows and tendency to over forecast or under forecast.

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$$\text{BIAS} = \frac{\text{Sum of forecast error for all periods}}{\text{No. of periods}}$$

BIAS indicates the directional tendency of forecast errors. If the forecast repeatedly overestimates actual demand Bias will have +ve value & underestimation will be indicated by a -ve bias.

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Some exceptional cases in LPP →

1) **Multiple optimal solutions** → for this

i) The objective function, when plotted should be parallel to a constraint that forms the boundary (edge) of the feasible region. ie. slope of objective function is same as that of one of the binding constraints / Active constraint.

2) The constraint should form a boundary of the feasible region in the direction of the optimal movement of the objective function line ie. the constraint should be an active constraint.

2) **Unbounded Solution** → It exists when an L.P. Problem has no limit on the constraints ie. the feasible region is not bounded in any respect. Value of the objective function may be finite or infinite. When it is infinite. When it is infinite, the problem is said to be have unbounded solution.

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3) <u>Infeasible solution</u> → Infeasibility is a condition that exists when there is a no solution to an LPP that satisfies all the constraints & non negativity restrictions. It means that the constraints in the problem are conflicting & inconsistent. Infeasibility depends solely on the constraints & has nothing to do with the objective function.